BetterTimes

Privacy-assured Outsourced Multiplications for Additively Homomorphic Encryption on Finite Fields

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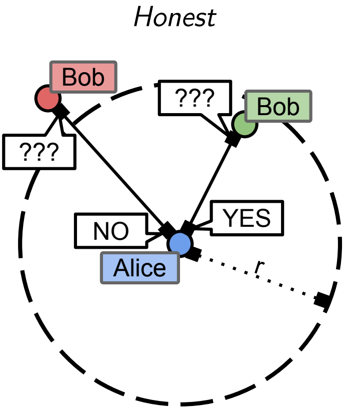

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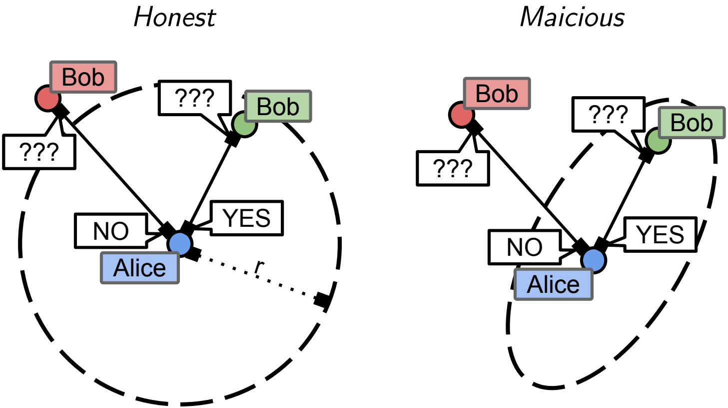

Arithmetic formulas

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  - Privacy-preserving location proximity
  - Privacy-preserving auctioning and bartering systems
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- Common assumption is honest-but-curious
- Many current solutions suffer
  - Face recognition: Sadeghi et al. 2009, Erkin et al. 2009
  - Location proximity: Zhong et al. 2007, Sedenka and Gasti 2014, Hallgren et al. 2015

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  - Two-party setting
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  - Goal
    - Bob learns nothing
    - Alice learns *at most* the intended output

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  - *Blinding*: given $\mathcal{M}^{\mathcal{U}}$ uniformly random distribution in $\mathcal{M} \setminus \{0\}$
    - $[\![m]\!] \oplus [\![b]\!] = [\![r]\!]$, with $b, r \in \mathcal{M}^{\mathcal{U}}$
    - $[\![m]\!] \odot [\![b]\!] = [\![r]\!]$, with $b, r \in \mathcal{M}^{\mathcal{U}}$

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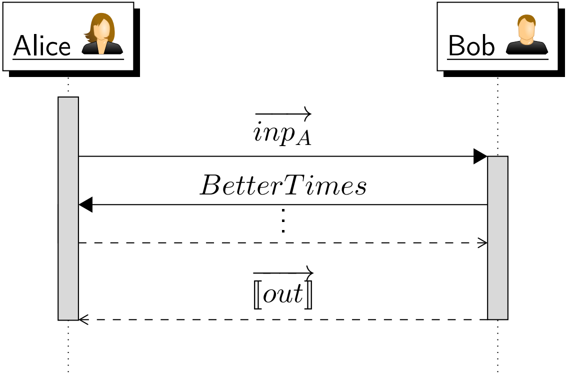
  - Multiplication *with one known plaintext*
- The only thing we need to add is $[\![m_1]\!] \odot [\![m_2]\!] = [\![m_1 \cdot m_2]\!]$
- We solve this using *outsourcing* these multiplications through a novel protocol called *BetterTimes*.

Communication Overview

- In our setting, protocols follow the form
  - Alice initiates the protocol
  - Bob sees only encrypted data (he can't decrypt)
  - Possibly there are more round trips to finish the computation
  - Bob responds with the final result



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5. Bob computes $[\![a]\!]$ using all of $[\![x']\!]$, $[\![y']\!]$, $[\![z']\!]$ and $[\![a']\!]$

BetterTimes communication

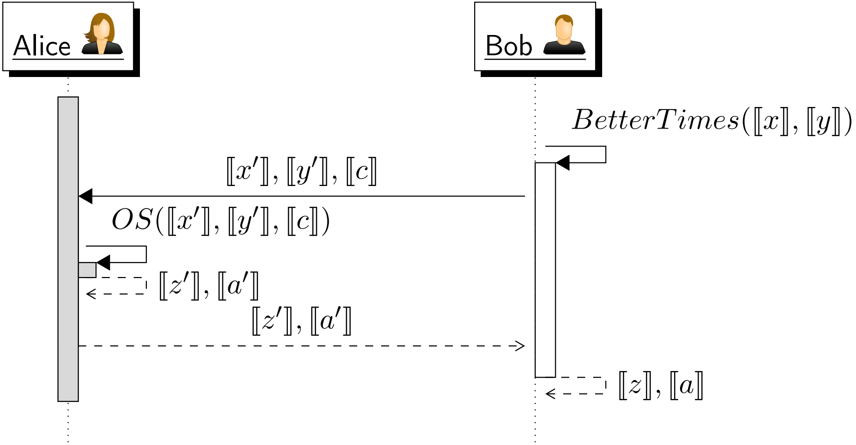

Figure: Visualization of the attested multiplication protocol

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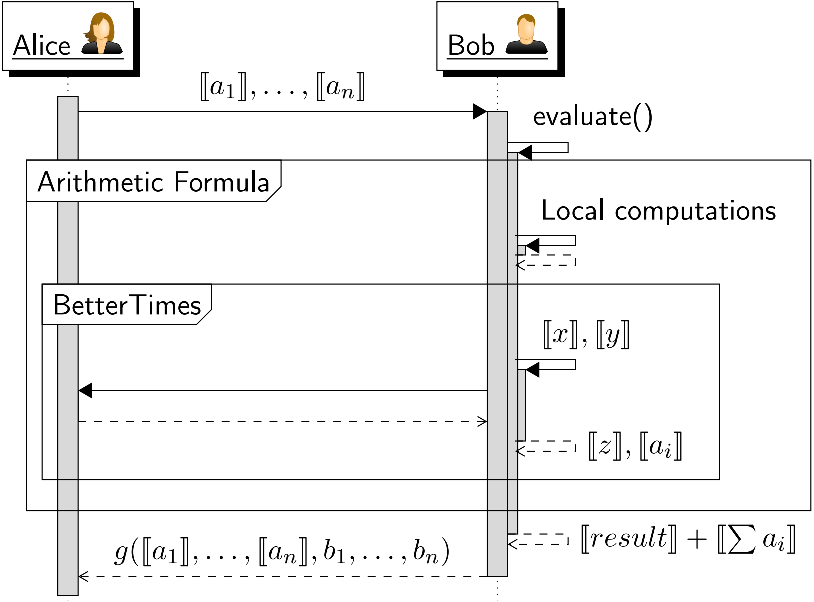
- When Bob has computed the final result $[\![result]\!]$, he sends $[\![result]\!] + \sum a_i$ to Alice, where a_i is the assurance value corresponding to each outsourced multiplication.
- Alice receives the correct output if and only if she computed all outsourced multiplications honestly, and a uniformly random value otherwise

Private Evaluation of Arithmetic Formula



Proof Outline for Privacy of Arbitrary Formula

Our Privacy definition follows the standard framework for secure multi-part computation (Lindell and Pinkas 2008)

Theorem

For a fixed but arbitrary arithmetic formula $g(\overrightarrow{x}, \overrightarrow{y})$ *represented by a recursive instruction* $\iota \in$ ***Ins****, for every adversary* $\mathcal{A}$ *against the protocol* π *resulting from* $evaluate(\iota)$*, there exist a simulator* $\mathcal{S}$ *such that:*

$$\{\mathsf{IDEAL}_{g,\mathcal{S}(s)}(\overrightarrow{x}, \overrightarrow{y})\} \overset{c}{\equiv} \{\mathsf{REAL}_{\pi,\mathcal{A}(s)}(\overrightarrow{x}, \overrightarrow{y})\}$$

where $\overset{c}{\equiv}$ *denotes computational indistinguishability of distributions.*

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- The full proof is given in the paper
- The theorem implicates that **any protocol** evaluating arithmetic formulas as defined in the paper can be evaluated in the presence of a malicious adversary while preserving privacy

Benchmarks

- Performed benchmarks on prototype implementation in python
- Comparing to outsourced multiplications secure only against honest adversaries

Table: Times (in milliseconds) for outsourced multiplication

Plaintext space	Time (in milliseconds)					
	1024 bits			2048 bits		
	This approach	Naive approach	Extra work	This approach	Naive approach	Extra work
2^2	6.286	4.016	56.52%	29.686	19.458	52.56%
2^8	6.400	4.017	59.32%	30.052	19.484	54.24%
2^{16}	6.432	4.148	55.06%	30.188	19.574	54.22%
2^{24}	6.538	4.100	59.46%	30.578	19.801	54.43%

- Benchmarks show that our more secure approach costs about 53-60% extra work for a multiplication

Protocols that can be secured with BetterTimes

- Several existing works can use the proposed approach to increase protection against malicious attackers
  - Privacy-preserving face recognition: Sadeghi et al. 2009, Erkin et al. 2009
  - Privacy-preserving location proximity: Zhong et al. 2007, Sedenka and Gasti 2014, Hallgren et al. 2015

Conclusions

- Presented BetterTimes
- Using BetterTimes one can compute any arithmetic formula in the presence of a malicious Alice
- The overhead, compared to protection against honest adversaries, is about 55%
  - Of each multiplication, not of the formula as a whole
  - Usually the number of multiplications is minimized, as additions are cheap with additively homomorphic encryption

Thank you for your attention!

Questions?